

TDC Odd Semester Exam., 2020
held in July, 2021

PHYSICS

(Honours)

(1st Semester)

Course No. : PSHH-102

(Mathematical Physics—I)

Full Marks : 35

Pass Marks : 12

Time : 2 hours

*The figures in the margin indicate full marks
for the questions*

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) What is meant by curvilinear coordinate? Define orthogonal curvilinear coordinates. 2+1=3
- (b) Find the expressions for length and volume elements in orthogonal curvilinear coordinate system. 4

2. (a) What are meant by right-handed and left-handed Cartesian coordinate systems? 2
- (b) Find the expression of unit vectors in curvilinear coordinate system. Show that the unit vectors of cylindrical coordinate system are mutually orthogonal to each other. 2+3=5

UNIT—II

3. (a) Define vector triple product. Show that vector triple product is not associative. 1+2=3
- (b) Show that

$$[\vec{a}, \vec{b}, \vec{c}], [\vec{c}, \vec{a}, \vec{b}], [\vec{a}\vec{b}\vec{c}]$$
 4
4. (a) Define the divergence of a vector point function. Interpret its physical meaning. 2+3=5
- (b) If $\vec{v} = x^2z\hat{i} + 2y^2z^2\hat{j} + xy^2z\hat{k}$, then find $\vec{\nabla} \cdot \vec{v}$ at the point (1, -1, 1). 2

UNIT—III

5. (a) Show that the eigenvalues of a Hermitian matrix are all real. 3

(3)

(b) Show that any two given vectors corresponding to two distinct eigenvalues of a Hermitian matrix are orthogonal. 4

6. (a) Define unitary matrix. 1

(b) Show that the following matrix is unitary : 3

$$\frac{1}{\sqrt{2}} \quad \frac{i}{\sqrt{2}}$$

$$\frac{i}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}$$

(c) Find the inverse of the following matrix : 3

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 7 & 9 \end{matrix}$$

UNIT—IV

7. (a) Obtain the relation between beta and gamma functions : 3

$$(m, n) = \frac{(m) (n)}{(m+n)}$$

(b) Prove that $(n-1) \Gamma(n) = \Gamma(n)$. 2

(c) Evaluate $\int_0^1 \sqrt[4]{x} e^{-\sqrt{x}} dx$. 2

(4)

8. (a) Show that

$$\int_0^1 \frac{y^{m-1}}{(1-y)^{m+n}} dy = \frac{(m) (n)}{(m+n)} \quad 3$$

(b) Prove that

$$1 \cdot 3 \cdot 5 \cdots (2n-1) = \frac{2^n}{\sqrt{\pi}} \Gamma\left(n + \frac{1}{2}\right) \quad 4$$

UNIT—V

9. (a) What is Fourier series? Evaluate the coefficients of Fourier series. 5

(b) State the Dirichlet's conditions associated with the Fourier series. 2

10. (a) Find the Fourier series of $f(x) = x$ for $0 < x < 2$. 3

(b) Use Fourier theorem to analyze the square wave in terms of its components. 4
