**2021/TDC(CBCS)/EVEN/SEM/
MTMSEC–401T/125 (A/B/C)**

**TDC (CBCS) Even Semester Exam.,
September—2021**

MATHEMATICS

(4th Semester)

Course No. : MTMSEC–401T

Full Marks : 50
Pass Marks : 20

Time : 3 hours

The figures in the margin indicate full marks for the questions

Candidates have to answer from *either* Option—A *or* Option—B *or* Option—C

OPTION—A

Course No. : MTMSEC–401T (A)

(Graph Theory)

SECTION—A

Answer any *fifteen* as directed : 1×15=15

1. Define simple graph.

2. If is a ring sum of two graphs and G is a graph, then G G _____.

(Fill in the blank)

3. A graph containing m edges can be decomposed in _____ different ways into pairs of subgraphs g_1, g_2.

(Fill in the blank)

4. Define complete graph.

5. Define pseudo-graph.

6. Define self-loop in a graph.

7. Define bridge in a graph.

8. Define block of a graph.

9. Define center of a graph.

10. Any graph without cycle is a tree/forest.

(Choose the correct option)

11. Define cut-point of a graph.

12. How many points in every nontrivial connected graph are not cut-points?

13. When are two graphs G and G said to be isomorphic?

14. A closed walk in which no vertex appears more than once is called a _____.

(Fill in the blank)

15. Define path.

16. A connected graph G is an Euler graph if and only if it can be decomposed into _____.
(Fill in the blank)

17. Define degree of a vertex in a graph.

18. Define weighted graph.

19. Define planar graphs.

20. The complete graph of five vertices is _____.
(Fill in the blank)

21. Every 3-connected planar graph is uniquely embeddable on the sphere/circle.
(Choose the correct option)

22. Every maximal outerplanar graph G with p points has _____ lines.
(Fill in the blank)

23. A graph is planar if and only if each of its blocks is planar.
(Write True or False)

24. If G be a maximal outerplanar graph with p 3 vertices all lying on the exterior face, then G has _____ interior faces.
(Fill in the blank)

25. Define Hamiltonian cycle.

26. Define shortest-path problem.

27. Define algorithm.

28. Fortran is a _____ language.
(Fill in the blank)

29. Define Dijkstra's algorithm.

30. What is an adjacency matrix?

Section—B

Answer any *five* questions : 2×5=10

31. State Königsberg bridge problem.

32. When is a graph G said to have been decomposed into two subgraphs g_1 and g_2?

33. Prove that every nontrivial tree has at least two points.

34. Let v be a point of a connected graph G, u and w distinct from v such that v is on every u–w path. Show that v is a cut-point of G.

35. If a graph has exactly two vertices of odd degree, then show that there must be a path joining these two vertices.

36. Define Hamiltonian circuit.

37. Prove that a graph G is outerplanar if each of its blocks is outerplanar.

38. State Kuratowski's theorem.

39. Describe travelling salesman's problem.

40. Write down the names of two algorithms which are used for solving the shortest-path problem.

SECTION—C

Answer any *five* questions : 5×5=25

41. Prove that a graph is bipartite iff it contains no odd cycle.

42. Define degree of a vertex in a graph. Prove that the number of vertices of odd degree in a graph is always even.

43. Prove that a graph H is the block of some graph if and only if every block of H is complete.

44. Prove that every tree has a center consisting of either one point or two adjacent points.

45. Prove that a connected graph G is an Euler graph if and only if all the vertices of G are of even degree.

46. Let G be a connected labeled graph with adjacency matrix A. Then show that all cofactors of the matrix M are equal and their common value is the number of spanning trees of G.

47. Prove that a connected planar graph with n vertices and e edges has $e \quad n \quad 2$ regions.

48. Prove that a graph is planar if and only if it has no subgraph homomorphic to K_5 or $K_{3, 3}$.

49. Prove that travelling salesman's problem is NP complete.

50. Describe the Floyd-Warshall algorithm.

OPTION—B

Course No. : MTMSEC–401T (B)

(Special Functions)

SECTION—A

Answer any *fifteen* as directed : 1×15=15

1. What is Legendre's differential equation?

2. Write the general solution of Legendre's equation.

3. State Laplace's first integral for $P_n(x)$.

4. What is the generating function of Legendre's polynomial?

5. Write down the number of terms in $P_n(x)$ if n is odd integer.

6. If we set $n \quad 0$ in Legendre's equation, then what is y in terms of x?

7. State orthogonal properties of Legendre's polynomials.

8. What is Bessel's differential equation?

9. What is the solution of Bessel's differential equation for $n \quad 0$ in expansion form?

10. Write Bessel's function of first kind of order n.

11. Write the solution of Bessel's equation for $n \quad 1$ in expansion form.

12. When n is a positive integer then $J_{\ n}(x) \quad$ ______.

(Fill in the blank)

13. Evaluate $L^{\ 1}\ \dfrac{1}{(s \quad 1)^2}$.

14. State convolution theorem.

15. If $L\{F(t)\} \quad f(s)$, then find $L\{F(2t)\}$.

16. Evaluate $L^{\ 1}\ \dfrac{1}{(2s \quad 5)}$.

17. State second shifting theory of Laplace transformation.

18. Does $L\ \dfrac{\cos at}{t}$ exist?

19. If $L\{F(t)\} \quad f(s)$, then what is $L\{F\ (t)\}$?

20. If $L\{F(t)\} \quad f(s)$, then what is $L\{F\ \ (t)\}$?

21. Write $L \ \frac{F(t)}{t}$ if $L\{F(t)\} \quad f(s)$.

22. If $L\{F(t)\} \quad f(s)$, then find $L\{t^2 F(t)\}$.

23. If $L\{Y(t)\} \quad y(s)$, then find the solution of

$$\frac{dy}{dt} \quad 2y \quad 4, \qquad y(0) \quad 1$$

24. Given the differential equation

$$(D^2 \quad 9)y \quad \cos 2t$$

Write Laplace transform of the given equation. (No need to solve the equation)

25. If $F\{F(x)\} \quad f(s)$, then find $F\{F(a \quad x)\}$.

26. Is $\ _0 \frac{\sin ax}{x} dx \quad \frac{}{2}, \ a \quad 0$?

(Write True or False)

27. Write infinite Fourier transform of $f(x)$.

28. Define Parseval's identity for Fourier transform.

29. If $F(F(x)) \quad f(s)$, then what is $F\{F\ (x)\}$?

30. The value of the integral $\ _0\, e^{\ ax} \cos bx\, dx$ ______.

(Fill in the blank)

SECTION—B

Answer any *five* questions : 2×5=10

31. If $P_n(x)$ be a Legendre's polynomial, then show that $P_n(1) \quad 1$.

32. Find the last term in $P_n(x)$ when n is even integer.

33. If $J_n(x)$ is a Bessel's function of order n, then prove that

$$x \ \ J_n(x) \quad nJ_n(x) \quad xJ_{n \ \ 1}(x)$$

34. Prove that $J_n(x) \quad 0$ has no repeated roots except at $x \quad 0$.

35. Prove that $L(\cos at) \quad \dfrac{s}{s^2 \quad a^2}$ if $L\{F(t)\} \quad f(s)$.

36. Find $L^{\ 1}\left[\dfrac{1}{s \quad 2} \quad \dfrac{2}{s \quad 5} \quad \dfrac{6}{s^4}\right]$ where $L^{\ 1}$ is inverse Laplace transform.

37. Using Laplace's transform, find the solution of $\dfrac{d^2y}{dt^2} \quad 25y \quad 0$ where $y(0) \quad 2,\ y\ (0) \quad 0$.

38. Using Laplace's transform, solve $\dfrac{d^2y}{dx^2} \quad y \quad \cos x$ where $y(0) \quad 0,\ y\ (0) \quad 0$.

39. Define infinite Fourier cosine and sine transform of $f(x)$.

40. Given function $F(t) \quad \begin{cases} e^{\ xt}\ (t), & t \quad 0 \\ 0, & t \quad 0 \end{cases}$

evaluate $F\{F(t)\}$.

SECTION—C

Answer any *five* questions : 5×5=25

41. Prove that, when n is positive integer

$$P_n(x) \quad \frac{1}{\ } \ _0 \frac{d}{\{x \quad \sqrt{x^2 \quad 1}\cos \quad \}^{n \quad 1}}$$

42. Show that $P_n(x)$ is the coefficient of z^n in the expansion of $(1 \quad 2xz \quad z^2)^{\ ½}$.

43. Show that

$$P_n \quad \frac{1}{2^n\, n!}\frac{d^n}{dx^n}(x^2 \quad 1)^n$$

44. Show that—

(i) $J_{\ ½}(x) \quad \sqrt{\dfrac{2}{x}}\cos x$

(ii) $J_{½}(x) \quad \sqrt{\dfrac{2}{x}}\sin x$

45. *(a)* Determine Laplace's transform of $2e^{2t} \quad 4e^{4t}$. 2

(b) Given function

$$f(t) \quad \begin{cases} 0, & t \quad 1 \\ t \quad 1, & 1 \quad t \quad 2 \\ 1, & t \quad 2 \end{cases}$$

Find $L\{f(t)\}$, where L is Laplace transform. 3

46. *(a)* Find the inverse Laplace transform of $\dfrac{3s \quad 7}{s^2 \quad 2s \quad 3}$. 3

(b) Prove that the inverse Laplace transform of $\frac{1}{s^2(s^2 \quad 1)}$ is $(t \quad \sin t)$. 2

47. Using Laplace transform, solve

$$y \ (t) \quad y(t) \quad t$$

given that $y\,(0) \quad 1,\ y(\ ) \quad 0.$

48. Solve by using Laplace's transform

$$\frac{d^2y}{dt^2} \quad 9y \quad \cos 2t$$

if $y(0) \quad 1,\ y \ \frac{}{2} \qquad 1,\ y\,(0) \quad a.$

49. Find the Fourier sine and cosine transform of $(2e^{\ 5x} \quad 5e^{\ 2x})$.

50. Using Fourier transform, prove that

$$_0 \frac{\cos \ \ x}{1 \quad ^2} d \qquad \frac{}{2} e^{\ x},\ x \quad 0$$

OPTION—C

Course No. : MTMSEC–401T (C)

(Vector Analysis)

SECTION—A

Answer any *fifteen* questions : 1×15=15

1. Under what condition three non-parallel and non-null vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar?

2. Write the value of $\hat{i} \quad (\hat{j} \quad \hat{k})$.

3. 'The scalar triple product does not depend on the position of dot and cross.' State True or False.

4. $(\vec{a} \quad \vec{b}) \quad \vec{c} \quad ?$

5. Write the vector equation of a line through the point $\vec{a}$ and parallel to the vector $\vec{b}$.

6. Write the vector equation of a sphere, whose center is at the origin and whose radius is a.

7. If $\underset{t \quad a}{\text{Lt}} \vec{f}(t) \quad L$, then $\underset{t \quad a}{\text{Lt}} \left| \vec{f}(t) \right| \quad ?$

8. If $\vec{r}\ \ (\sin t)\hat{i}\ \ (\cos t)\hat{j}\ \ t\hat{k}$, then find $\dfrac{d\vec{r}}{dt}$.

9. $\dfrac{d}{dt}(\vec{a}\ \ \vec{b})\ \ ?$

10. Show that $\vec{r}\ \ \dfrac{d\vec{r}}{dt}$ if

$$\vec{r}\ \ (\cos\ \ t)\hat{i}\ \ (\sin\ \ t)\hat{j}$$

11. What is the necessary and sufficient condition for a vector $\vec{f}(t)$ to have a constant direction?

12. If $\vec{a}\ \ 5t\hat{i}$ and $\vec{b}\ \ t^2\hat{j}$, then find $\dfrac{d}{dt}(\vec{a}\ \ \vec{b})$.

13. If $f(x, y, z)\ \ x^3\ \ y^3\ \ z^3$, then find $\vec{\ }f$.

14. If $\vec{f}\ \ x^2y\hat{i}\ \ 2xz\hat{j}\ \ 2yz\hat{k}$, then find div $\vec{f}$.

15. If $\vec{a}$ is a constant vector, then what is div $\vec{a}$?

16. If $\vec{r}\ \ x\hat{i}\ \ y\hat{j}\ \ z\hat{k}$, then find curl $\vec{r}$.

17. Define a solenoidal vector.

18. When is a vector $\vec{f}$ said to be irrotational?

19. Evaluate $(e^t\hat{i}\ \ e^{2t}\hat{j}\ \ t\hat{k})\,dt$.

20. If $\vec{f}(t)\ \ (1\ \ t^2)\hat{i}\ \ 2t^3\hat{j}\ \ 3\hat{k}$, then find $\vec{f}(t)\,dt$.

21. $\vec{u}\ \ \dfrac{d\vec{v}}{dt}\ \ \dfrac{d\vec{u}}{dt}\ \ \vec{v}\ \ dt\ \ ?$

22. Solve : $\dfrac{d^2\vec{r}}{dt^2}\ \ \vec{0}$

23. Find $(\vec{a}\ \ \vec{b})\,dt$ if $\vec{a}\ \ 3\hat{i}\ \ 3\hat{j}\ \ 6\hat{k}$ and $\vec{b}\ \ \hat{i}\ \ \hat{j}\ \ 2\hat{k}$.

24. Find $_0^{/2}(\sin t\hat{i}\ \ \cos t\hat{j})\,dt$.

25. Write the expressions for tangential and normal components of acceleration.

26. State the principle of work.

27. State the principle of conservation of momentum.

28. State the principle of conservation of energy.

29. If velocity $\vec{v}$ of a particle is given by $\vec{v} \quad 2t\hat{i} \quad \hat{j} \quad 4t^2\hat{k}$, then what is its acceleration?

30. What do you mean by conservative force?

SECTION—B

Answer any *five* questions : 2×5=10

31. Prove that

$$\vec{a} \quad (\vec{b} \quad \vec{c}) \quad \vec{b} \quad (\vec{c} \quad \vec{a}) \quad \vec{c} \quad (\vec{a} \quad \vec{b}) \quad \vec{0}$$

32. Find the vector equation of a line passing through the points (1, 0, 1) and (0, 2, 2).

33. Show that derivative of a constant vector is the zero vector.

34. If $\vec{a} \quad 7t^2\hat{i} \quad t\hat{j} \quad t^3\hat{k}$ and $\vec{b} \quad 2t\hat{i} \quad 5t^2\hat{j}$, then find $\frac{d}{dt}(\vec{a} \quad \vec{b})$.

35. If $f(x, y, z) \quad 3x^2y \quad y^3z^2$, then find $\vec{\ }f$ at the point (1, 1, 2).

36. If $\vec{f} \quad z\hat{i} \quad x\hat{j} \quad y\hat{k}$, then show that

$$\text{curl curl } \vec{f} \quad \vec{0}$$

37. Find the value of $\vec{r}$ satisfying the equation

$$\frac{d^2\vec{r}}{dt^2} \quad t\vec{a} \quad \vec{b}$$

where $\vec{a}$ and $\vec{b}$ are constant vectors.

38. If $\vec{r} \quad t\hat{i} \quad t^2\hat{j} \quad (t \quad 1)\hat{k}$ and $\vec{s} \quad 2t^2\hat{i} \quad 6t\hat{k}$, then evaluate $_0^2(\vec{r} \quad \vec{s})\,dt$.

39. A particle moves in such a way that its velocity at a point is given by

$$\vec{v} \quad \cos nt\,\hat{i} \quad \sin nt\,\hat{j}$$

Show that $\vec{v} \quad \vec{a} \quad n\hat{k}$, where $\vec{a}$ is the acceleration of the particle at that point. (n is a constant)

40. Define impulse and show that

impulse = change in momentum

SECTION—C

Answer any *five* questions : 5×5=25

41. *(a)* Show that the vectors $\vec{a} \quad \vec{b}, \vec{b} \quad \vec{c}$ and $\vec{c} \quad \vec{a}$ are coplanar if $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar. 3

(b) If $\hat{i}, \hat{j}$ and $\hat{k}$ have usual meanings, then prove that $\hat{i} \quad \hat{j}, \hat{j} \quad \hat{k}$ and $\hat{k} \quad \hat{i}$ are coplanar. 2

42. Find the vector equation of a plane through a given point $\vec{a}$ and parallel to the vectors $\vec{b}$ and $\vec{c}$.

43. *(a)* If $\vec{r} \quad 5t^2\hat{i} \quad t\hat{j} \quad t^3\hat{k}, \vec{s} \quad (\sin t)\hat{i} \quad (\cos t)\hat{j}$, then find $\frac{d}{dt}(\vec{r} \quad \vec{s})$. 3

(b) If $\vec{x} \quad e^{3t}\hat{a} \quad e^{\ 3t}\hat{b}$ where $\hat{a}$ and $\hat{b}$ are constant vectors, then show that

$$\frac{d^2\vec{x}}{dt^2} \quad 9\vec{x} \quad \vec{0}$$ 2

44. If $\vec{r} \quad (a\cos t)\hat{i} \quad (a\sin t)\hat{j} \quad (at\tan \quad)\hat{k}$, then prove that

$$\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3} \qquad a^3 \tan$$

45. Prove that $\text{div curl}\ \vec{A} \quad 0$ where $\vec{A} \quad A_1\hat{i} \quad A_2\hat{j} \quad A_3\hat{k}$.

46. *(a)* Show that $\ ^{\rightarrow} r^n \quad nr^n \ ^2\vec{r}$. 3

(b) If $\vec{f} \quad (x \quad y \quad 1)\hat{i} \quad \hat{j} \quad (x \quad y)\hat{k}$, then prove that $\vec{f} \quad \text{curl}\ \vec{f} \quad 0$. 2

47. Evaluate $\ _0^2\vec{a} \ (\vec{b} \quad \vec{c})\, dt$ where $\vec{a} \quad t\hat{i} \quad 3\hat{j} \quad 2t\hat{k}, \quad \vec{b} \quad \hat{i} \quad 2\hat{j} \quad 2\hat{k}$ and $\vec{c} \quad 3\hat{i} \quad t\hat{j} \quad \hat{k}$.

48. Evaluate $\ _1^2\vec{r} \ \frac{d^2\vec{r}}{dt^2}\, dt$ where $\vec{r} \quad 2t^2\hat{i} \quad t\hat{j} \quad 3t^3\hat{k}$.

49. Prove the principle of conservation of linear momentum.

50. Prove the principle of work.

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