2020/TDC/ODD/SEM/MTMP–301/264

TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Pass)

(3rd Semester)

Course No. : MTMP–301

(Differential Calculus and Integral Calculus)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

GROUP—A

(Differential Calculus)

(*Marks* : 30)

UNIT—I

1. (a) If

$$f(x) \quad 3 \quad 2x, \quad \frac{3}{2} \quad x \quad 0$$
$$3 \quad 2x, \ 0 \quad x \quad \frac{3}{2}$$

then show that $f(x)$ is continuous at $x \quad 0$. 2

(b) If f is differentiable at c and $f(c) \quad 0$, then show that

$$\frac{1}{f} \ (c) \quad \frac{f \ (c)}{\{f(c)\}^2}$$ 3

(c) State and prove Leibnitz's theorem for the nth derivative of the product of two functions. 1+4=5

2. (a) Show that the function $f(x) \quad |x \quad 1|$ is not differentiable at $x \quad 1$. 2

(b) Using Cauchy's criterion, show that $\underset{x \quad 0}{\text{Lt}} \sin\frac{1}{x}$ does not exist. 3

(c) If $y \quad \sin(m \sin^{\ 1} x)$, then show that

(i) $(1 \quad x^2)y_2 \quad xy_1 \quad m^2 y \quad 0$

(ii) $(1 \quad x^2)y_{n \ 2} \quad (2n \quad 1)xy_{n \ 1}$
$(m^2 \quad n^2)y_n \quad 0$ 2+3=5

UNIT—II

3. (a) Evaluate : 3

$$\underset{x \quad 0}{\text{Lt}} \frac{\cot x \quad \frac{1}{x}}{x}$$

(b) State and prove Rolle's theorem. 4

(c) Prove that the greatest rectangle to be inscribed in a circle is a square. 3

4. (a) From the relation

$$f(x) \quad f(0) \quad xf\ (0) \quad \frac{x^2}{\lfloor 2} f\ (\ x)$$

where 0 1, show that

(i) $\log(1 \quad x) \quad x \quad \frac{x^2}{2}$, if $x \quad 0$

(ii) $\cos x \quad 1 \quad \frac{x^2}{2}$, if $0 \quad x \quad \frac{}{2}$ 2+2=4

(b) Expand $\sin x$ in Maclaurin's infinite series. 3

(c) Show that $12(\log x \quad 1) \quad x^2 \quad 10x \quad 3$ is maximum when $x \quad 2$ and minimum when $x \quad 3$. 1½+1½=3

UNIT—III

5. (a) If

$$u \quad \log \frac{x^4 \quad y^4}{x \quad y}$$

then show that $x\frac{u}{x} \quad y\frac{u}{y} \quad 3$ 3

(b) If $z \quad e^{xy^2}$, $x \quad t\cos t$, $y \quad t\sin t$, then evaluate

$$\frac{dz}{dt} \text{ at } t \quad \frac{}{2}$$ 3

(c) Prove that the curve

$$\frac{x}{a}^{\,n} \quad \frac{y}{b}^{\,n} \quad 2$$

touches the straight line $\frac{x}{a} \quad \frac{y}{b} \quad 2$ at the point (a, b) whatever be the value of n. 4

6. (a) State and prove Euler's theorem on the function which is homogeneous of degree n in x, y. 1+3=4

(b) If $u \quad \log(x^3 \quad y^3 \quad z^3 \quad 3xyz)$, then show that

$$\frac{u}{x} \quad \frac{u}{y} \quad \frac{u}{z} \quad \frac{3}{x \quad y \quad z}$$ 3

(c) Prove that in the curve $by^2 \quad (x \quad a)^3$ the square of the subtangent varies as the subnormal. 3

GROUP—B

(Integral Calculus)

(*Marks* : 20)

UNIT—IV

7. *(a)* Prove that

$$\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$$

if $f(2a-x) = f(x)$. 2

(b) Show that

$$\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}dx = \frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$$ 4

(c) Obtain the reduction formula for

$$\int_0^{\pi/2} \sin^n x\,dx$$ 4

8. *(a)* State the fundamental theorem of integral calculus. 2

(b) Evaluate : 3

$$\underset{n\to\infty}{\text{Lt}}\left[\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right]$$

(c) Obtain the reduction formula for

$$\int_0^{\pi/4} \tan^n x\,dx$$

and hence find $\int_0^{\pi/4} \tan^6 x\,dx$. 3+2=5

UNIT—V

9. *(a)* Find the area bounded by the curve $r = a(1+\cos\theta)$. 5

(b) Find the surface area generated by the curve $y = a\sin\frac{x}{a}$ extended from $x = 0$ to $x = \pi a$ on rotating about x-axis. 5

10. *(a)* Find the length of the curve in the first quadrant

$$x^{2/3} + y^{2/3} = a^{2/3}$$ 5

(b) Show that the volume generated by revolution of the curve $y(a^2+x^2) = a^3$ about its asymptote is $\frac{1}{2}\pi^2 a^3$. 5

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