TDC Odd Semester Exam., 2020
held in July, 2021

MATHEMATICS

(Pass)

(1st Semester)

Course No. : MTMP–101

(Classical Algebra and Trigonometry)

Full Marks : 50
Pass Marks : 17

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

GROUP—A

(Classical Algebra)

(*Marks* : 30)

UNIT—I

1. (a) If $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$, then show that

$$\operatorname{adj}(\operatorname{adj} A) = A$$ 2

(b) Find the rank of the matrix

$$A = \begin{pmatrix} 1 & -2 & 4 & 0 \\ 0 & 3 & 2 & 1 \\ 1 & -2 & 4 & 0 \\ 0 & 3 & 2 & 1 \end{pmatrix}$$ 4

(c) Solve the following by matrix inversion method : 4

$$x + y + z = 6$$
$$x - y + z = 2$$
$$2x - y + 3z = 9$$

2. (a) If A and B are two invertible square matrices of the same order, then prove that AB is also invertible and that

$$(AB)^{-1} = B^{-1}A^{-1}$$ 1+2=3

(b) State and prove Jacobi's theorem. 4

(c) Reduce the following matrix to normal form : 3

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ -2 & -3 & -1 \end{pmatrix}$$

UNIT—II

3. *(a)* If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then find the value of

$$\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$$ 3

(b) Solve the following by Cardan's method : 4

$$x^3 - 12x + 65 = 0$$

(c) If $a_1, a_2, \cdots, a_n$ be n numbers in AP, then prove that

$$a_1 a_2 \cdots a_n < \left(\frac{a_1 + a_n}{2}\right)^n$$ 3

4. *(a)* Solve the equation

$$x^3 - 3x^2 - 6x + 8 = 0$$

given that the roots are in AP. 3

(b) If α, β, γ are the roots of the equation

$$x^3 - ax^2 + bx - c = 0$$

then find the equation whose roots are

$$\beta\gamma + \frac{1}{\alpha},\ \gamma\alpha + \frac{1}{\beta},\ \alpha\beta + \frac{1}{\gamma}$$ 4

(c) If $x + y + z = 1$, then prove that

$$(1-x)(1-y)(1-z) > 8xyz$$ 3

UNIT—III

5. *(a)* Prove that every convergent sequence is bounded. Give an example to show that the converse is not true. 3+1=4

(b) Show that the sequence $\{x_n\}$, where

$$x_n = \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$$

is convergent. 3

(c) Discuss the convergence of the series

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \cdots \text{ to } \infty$$ 3

6. *(a)* Show that the sequence

$$\sqrt{2},\ \sqrt{2+\sqrt{2}},\ \sqrt{2+\sqrt{2+\sqrt{2}}},\ \cdots$$

is convergent and hence find its limit. 3

(b) State and prove ratio test for convergence of an infinite series. 4

(c) Test the convergence of the series

$$\frac{1}{1.2^2} + \frac{1}{2.3^2} + \frac{1}{3.4^2} + \cdots \text{ to } \infty$$ 3

GROUP—B

(Trigonometry)

(*Marks* : 20)

UNIT—IV

7. *(a)* Prove that

$$\left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n=\cos n\theta+i\sin n\theta$$ 3

(b) Expand $\cos x$ in ascending powers of x. 4

(c) Prove that

$$\log(\alpha+i\beta)=\frac{1}{2}\log(\alpha^2+\beta^2)+i\tan^{-1}\left(\frac{\beta}{\alpha}\right)$$ 3

8. *(a)* Solve the equation

$$x^7+x^4+x^3+1=0$$ 3

(b) Prove that

$$\frac{1}{6}\sin^3 x=\frac{x^3}{3!}-\frac{x^5}{5!}(1+3^2)+\frac{x^7}{7!}(1+3^2+3^4)+\cdots$$ 4

(c) Show that i^i is a pure real number. Find its principal value. 3

UNIT—V

9. *(a)* State and prove Gregory's series. 4

(b) If $x+iy=\tan(\alpha+i\beta)$, then prove that

$$x^2+y^2-2y\coth 2\beta+1=0$$ 3

(c) Find the sum of the series

$$\sin\theta-\frac{1}{2}\sin 2\theta+\frac{1}{3}\sin 3\theta-\frac{1}{4}\sin 4\theta+\cdots \text{ to } \infty$$ 3

10. *(a)* If θ lies between 0 and $\pi/2$, then prove that

$$\tan^{-1}\left(\frac{1-\cos\theta}{1+\cos\theta}\right)=\tan^2\left(\frac{\theta}{2}\right)-\frac{1}{3}\tan^6\left(\frac{\theta}{2}\right)+\frac{1}{5}\tan^{10}\left(\frac{\theta}{2}\right)-\cdots \text{ to } \infty$$ 3

(b) If $\cosh x=\sec\theta$, then prove that

$$\tanh^2\left(\frac{x}{2}\right)=\tan^2\left(\frac{\theta}{2}\right)$$ 3

(c) Show that

$$\sin\alpha+\sin(\alpha+\beta)+\sin(\alpha+2\beta)+\cdots+\sin\{\alpha+(n-1)\beta\}=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\{\alpha+(n-1)\beta\}$$ 4

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